

1. Let X be a continuous random variable such that its density function is

$$f(x) = \begin{cases} k(x^2 + 1), & 0 < x < 1 \\ 0, & \text{elsewhere.} \end{cases}$$

- (a) Evaluate k .
(b) Find $F(x)$ and use it to evaluate $P(0.2 < X < 0.5)$.

2. We consider a sack of fruit which contains 3 oranges, 4 apples and 5 bananas. We select randomly 2 pieces of fruit. Find the probability that we obtain 1 oranges and 1 apples if:

- (a) the 2 pieces are selected at the same time.
(b) the 2 pieces are selected in succession with replacement.
(c) the 2 pieces are selected in succession without replacement.

Conclusion?

3. (a) How many three-digit numbers can be formed from the digits 0, 1, 2, 3, 4 and 5 if each digit can be used only once?
(b) How many of these are odd numbers?
(c) How many of these are even numbers?
(d) How many of these are greater than 450?
(e) How many of these are divisible by 5?
(f) *Bonus question.* How many of these are divisible by 3 and 5?

4. We consider tow bags such that the first bag contains 5 white balls and 6 black balls and the second bag contains 8 white balls and 7 black balls. We draw one ball from the first bag and we placed unseen in the second bag. After, we draw one ball from the second bag and we placed unseen in the first bag. Finally, we draw a ball from the first bag. Find the probability that the three drawn balls have the same color.

MARKS : 1. [25] 2. [25] 3. [30] 4. [20] Bonus question [10]

1.
 - (a) How many four-digit numbers can be formed from the digits 0, 1, 2, 3, 4, 5, 6 and 7 if each digit can be used only once?
 - (b) How many of these are even numbers?
 - (c) How many of these are odd numbers?
 - (d) How many of these are divisible by 5?
 - (e) How many of these contain the digit 2?
 - (f) How many of these contain the digits 3 and 5 at the same time?
 - (g) How many of these contain two odd digits and two even digits at the same time?

2. We consider three dice: one red, one blue and one green. We assume that
 - the red die is fair,
 - the blue die is loaded in such a way that an even number is twice as likely to occur as an odd number,
 - the green die is loaded in such a way that an odd number is twice as likely to occur as an even number.We toss the three dice. Find the probability that
 - (a) we obtain the same number on the three dice?
 - (b) we obtain two even numbers and one odd number on the three dice?
 - (c) we obtain the number 3 exactly two times on the three dice?
 - (d) *Bonus question.* we do not obtain the number 3?

3. We consider a bag which contains 5 red balls and 7 blue balls. We draw (without replacement) two balls from the bag. If the drawn balls have the same color then we put a new red ball in the bag, and if the drawn balls have different color then we put a new blue ball in the bag. Finally we draw one ball from the bag. Find the probability that the last drawn ball is red?

MARKS : 1. [40] 2. [30] 3. [30] Bonus question [10]

1. A fair die is tossed five times.
 - (a) Find the probability that we obtain the same number in the five tosses.
 - (b) Find the probability that we obtain five even numbers in the five tosses.
 - (c) Find the probability that we obtain two even numbers and three odd numbers in the five tosses.

2. From the set $\{1, 2, 3, 4, \dots, 144, 145\}$, we select in succession and without replacement three numbers. Find the probability that
 - (a) the product of the three selected numbers is even.
 - (b) the sum of the three selected numbers is even.

3. We consider a bag which contains 7 red balls and 9 blue balls. We draw (without replacement) one ball from the bag. If the drawn ball is red then we place two new blue balls in the bag, and if the drawn ball is blue then we place two new red balls in the bag. Finally we draw one ball from the bag.
 - (a) Find the probability that the two drawn balls have the same color?
 - (b) Given that the second drawn ball is red, find the probability that the first one is blue?

4. A die is loaded in such a way that an even number is thrice as likely to occur as an odd number. We consider the following experiment:

“First we toss the loaded die. If we obtain an even number then we flip a coin three times, otherwise we flip the coin four times”.

 - (a) Find the probability that we obtain exactly two heads.
 - (b) Find the probability that we obtain a number of heads equals to the number of tails.
 - (c) *Bonus question.* Find the probability that the number obtained on the die equals to the number of obtained heads.

MARKS : 1. [20] 2. [25] 3. [30] 4. [25] Bonus question [10]

1. A man comes home. He has 8 keys on his keychain, but he is drunk he can't remember which is the key to the front door. So he randomly chooses one key after the other until he picks the right one. Find the probability that the man picks the right key on the fifth try if

- (i) the man is so drunk that he is likely to pick the same key again even though he just tried it.
- (ii) the man is not so drunk so he tries a key only once.

Conclusion?

2. In an NBA (National Basketball Association) championship series, the team which wins four games out of seven will be the winner. Assume that Chicago Bulls has probability p of winning over Indiana Pacers, and that both teams face each other in the championship games.

- (a) Find the probability that Chicago Bulls will win the series in five games?
- (b) Find the probability that Chicago Bulls will lose the series in six games?
- (c) Find the probability that Chicago Bulls will win the series?

3. We consider two dice: one red and one blue. We assume that

- the red die is loaded in such a way that an even number is thrice as likely to occur as an odd number,
- the blue die is loaded in such a way that an odd number is thrice as likely to occur as an even number.

We toss the two dice five times.

- (a) Find the probability that we obtain the same even sum in the five tosses.
- (b) Find the probability that we obtain an even sum in the five tosses.
- (c) Find the probability that we obtain exactly three even sum in the five tosses.
- (d) Find the probability that we obtain an odd number of even sum in the five tosses.

4. We consider a bag which contains 5 red balls, 3 blues balls and 7 yellow balls. We also consider a loaded die in such a way that an even number is twice as likely to occur as an odd number. We consider the following two consecutive experiments:

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- We draw in succession and with replacement four balls from the bag.
 - We toss the die several times to obtain $n + 1$ times an even number for the first time, where n is the number of obtained blue balls in the first experiment.
- (a) Find the probability that we obtain exactly two blue balls in the first experiment.
 - (b) Find the probability that we obtain exactly two blue balls in the first experiment and that we toss the die four times in the second experiment.
 - (c) Find the probability that we toss the die four times in the second experiment.

Bonus question. We consider four dice, one red, one blue, one green and one yellow. We assume that:

- the red die is loaded in such a way that an even number is twice as likely to occur as an odd number,
- the blue die is loaded in such a way that an even number is thrice as likely to occur as an odd number.
- the green and the yellow dice are fair.

We toss the four dice. Find the probability that we obtain a same number a on the red and the green dice and a same number b on the blue and the yellow dice where $a \neq b$.

MARKS : 1. [15] 2. [25] 3. [30] 4. [30] 5. [10]

Byblos

Probability and Statistics
Test #1

Date: 23/03/2012
Duration: 2h

Name:

ID:

1. In a study of pleas and prison sentences, it is found that 45% of the subjects studied were sent to prison. Among those sent to prison, 40% chose to plead guilty. Among those not sent to prison, 55% chose to plead guilty. If a study subject is randomly selected and it is then found that the subject entered a guilty plea, find the probability that this person was not sent to prison.
2. Among a large group of patients recovering from shoulder injuries, it is found that 22% visit both a physical therapist and a chiropractor, whereas 12% visit neither of these. The probability that a patient visits a chiropractor exceeds by 0.14 the probability that a patient visits a physical therapist. Determine the probability that a randomly chosen member of this group visits a physical therapist.
3. A doctor is studying the relationship between blood pressure and heartbeat abnormalities in her patients. She tests a random sample of her patients and notes their blood pressures (high, low, or normal) and their heartbeats (regular or irregular). She finds that:
 - 14% have high blood pressure.
 - 22% have low blood pressure.
 - 15% have an irregular heartbeat.
 - Of those with an irregular heartbeat, one-third have high blood pressure.
 - Of those with normal blood pressure, one-eighth have an irregular heartbeat.
 - (a) What is the probability that a selected patient who has irregular heartbeat also has low blood pressure?
 - (b) What is the probability that a selected patient has a regular heartbeat and low blood pressure?
4. A garage door opener has a ten-digit keypad. Codes to the door must consist of 5 digits with no adjacent digits the same.
 - (a) How many codes are possible if no restriction?
 - (b) How many palindrome codes are possible? (A palindrome code is a code which reads the same from left to right as right to left)
 - (c) How many codes are possible if the first and the third digits are the same?
 - (d) How many codes are possible if the first and the last digits are the same?
5. You and 9 of your friends are at a restaurant where they serve 12 different meals. Suppose that you and your friends all order a meal and that the different meals are all equally likely to be chosen.
 - (a) What is the probability that no one orders the first meal on the list?
 - (b) What is the probability that at least two persons order the same meal?
 - (c) What is the probability that only two out of your 9 friends order the same meal as you?
 - (d) What is the probability that the 10 persons (you and your 9 friends) order exactly two meals from the list?

MARKS : 1. [20] 2. [20] 3. [20] 4. [25] 5. [25]

1. There is a new diagnostic test for testing quality of computer chip. It is designed to be used for a manufacture which has about 0.05% of defective chips. The test is not perfect but will detect a bad computer chip 95% of the time. It will, however, say that a good computer chip is bad about 4% of the time. If a computer chip produced by this manufacture is selected at random and the test indicates that it is bad, what is the probability that this selected computer chip is really a bad chip?
2. A real estate agent has 11 keys to open 10 new homes. Each key can open exactly one home except one master key which can open all the new homes. If 40% of these new homes are usually left unlocked, what is the probability that the real estate agent can get into a specific home if the agent selects 3 keys at random before leaving the office?
3. From the set $\{1, 2, \dots, 59, 60\}$ we select in succession and with replacement two numbers.
 - (a) Find the probability that the two selected numbers are the same?
 - (b) Find the probability that the first selected number is less than the second selected number?
 - (c) What will be the answer of the preceding question if the selection was in succession but without replacement? Explain.
4. A license plate in a certain state consists of 4 digits, not necessarily distinct, and 3 letters, also not necessarily distinct.
 - (a) How many distinct license plates are possible if no restriction?
 - (b) How many distinct license plates are possible if it must begin and terminate by a digit?
 - (c) How many distinct license plates are possible if it must begin and terminate by a letter?
 - (d) How many distinct license plates are possible if the three letters must appear next to each other?
 - (e) How many distinct palindrome license plates are possible?
(A palindrome license plate is a license plate that reads the same from left to right as right to left)
5. Let A and B be two events such that:

$$P(A) = 1/2 \text{ and } P(A|B) = P(A'|B') = 2/3$$

Find $P(B)$.

MARKS : 1. [20] 2. [25] 3. [25] 4. [30] 5. [10]